FTL leads to going backward in time

Now that we've covered the lead up, let us consider a general FTL drive in the context of special relativity. In special relativity we must assume that there is no preferred reference frame, so the FTL drive behaves identically (from the viewpoint of its passengers) no matter what its absolute speed is in the universe, assuming one could even define such a speed in a meaningful way.

We will use the Lorentz transformations derived earlier to calculate how long an FTL ship is away from Earth for a particular journey. The journey will consist of the following steps:

i At time $t = 0$ the ship leaves Earth (located at $x = 0$) for a stellar system *Hyperion* located a distance d_{Hyp} away, in the $+x$ direction, with its FTL drive.

ii The ship arrives at *Hyperion* at time t_{ii}.

iii The ship spends a certain amount of time, Δt_{iii} (as measured by an observer at rest on Earth), taking photographs and then accelerating to a slower-than-light speed v relative to the Earth, in the x direction (away from the Earth).

iv The ship leaves *Hyperion* for Earth using the same FTL drive it used earlier.

v The ship arrives back at Earth at time t_v.

Now, assume that the FTL drive has an effective normal space velocity of u. That is, the time it takes to cover a distance d is d/u, as measured by a clock which stayed behind and was at rest relative to the ship before the ship activated its drive. This time is not the time it takes for light from the arriving ship to arrive back at the clock which stayed behind, it's the true time taken by subtracting out the known time delay caused by the finite speed of light.

So, what is the total elapsed time, $t_v - t_i$? Let's set $t_i = 0$ for simplicity. We can set a couple of these other variables fairly easily:

$$\begin{aligned} t_{ii} &= d_{Hyp}/u \\ t_{iv} &= t_{ii} + \Delta t_{iii} \end{aligned} \tag{1}$$

The next step is not so simple. The ship is now moving under FTL drive with a speed u as measured in the *new* frame, the one moving at speed v relative to the Earth. In this frame, the distance from *Hyperion* to Earth is given by $d'_{Hyp} = d_{Hyp}/\gamma$ where γ was defined in the section about Lorentz transformations. So, in this new frame the ship will cover a distance and time interval:

$$\begin{aligned} \Delta x' &= -d_{Hyp}/\gamma \\ \Delta t' &= d_{Hyp}/(\gamma u) \end{aligned} \tag{2}$$

We now apply the Lorentz transformation to find the time elapsed in the frame of the Earth.

$$\Delta t = \gamma(\Delta t' + \frac{v\Delta x'}{c^2}) \tag{3}$$

This yields an elapsed time for the return journey in the Earth's frame of $\Delta t = d_{Hyp}(\frac{1}{u} - \frac{v}{c^2})$. Now, here's the crucial point. If $u > c$, this Δt can be negative for values of $v < c$. This can't happen if $u \leq c$. So, the time taken for the return journey can take negative time as measured by people on the Earth. The total time of the trip is then:

$$t_v = \frac{2d_{Hyp}}{u} + \Delta t_{iii} - \frac{d_{Hyp} v}{c^2} \tag{4}$$

For $v > 0$ and sufficiently large d it is possible for this sum to be negative, but only if $u > c$. So, we have derived the following result: if an FTL ship goes from point A to B, then accelerates away from A once it arrives at B, and activates the FTL drive again to return to A, it is entirely possible that it will arrive at A before it ever left.

Now, a *caveat*. This analysis is not valid if the FTL drive violates the "no preferred frame" postulate of special relativity. If the FTL drive behaves differently if moving than if at rest, it is possible to arrange things so that a time machine cannot be produced. For instance, if the FTL drive always causes the person using it to move at a speed u relative to the stationary frame of the cosmic microwave background, these time loops will not appear. This would violate the postulates of special relativity, but there is really no reason to believe that this violation would have been seen by experiments to date. In other words, there is room to fit such an FTL drive which remains consistent with all experiments performed to date.